Problem 13

Transform Eqs. (2) for the parallel circuit into a single second order equation.

Solution

Eqs. (2) are on page 360 in the textbook.

$$\frac{dI}{dt} = \frac{V}{L}$$

$$\frac{dV}{dt} = -\frac{I}{C} - \frac{V}{RC} \tag{2}$$

Solve this first equation for V

$$V = L\frac{dI}{dt}$$

and then plug it into the second equation.

$$\frac{d}{dt}\left(L\frac{dI}{dt}\right) = -\frac{I}{C} - \frac{1}{RC}\left(L\frac{dI}{dt}\right)$$

$$L\frac{d^2I}{dt^2} = -\frac{I}{C} - \frac{L}{RC}\frac{dI}{dt}$$

Divide both sides by L.

$$\frac{d^2I}{dt^2} = -\frac{I}{LC} - \frac{1}{RC}\frac{dI}{dt}$$

Bring all terms to the left side.

$$\frac{d^2I}{dt^2} + \frac{1}{RC}\frac{dI}{dt} + \frac{1}{LC}I = 0$$